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Mathematical Models of the Mother and Child Attachment System

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1 Background

Over the course of about the first eighteen months a baby develops an attachment style [1]. Once the child is around eighteen months this attachment style can be measured by observing the mother and child in a scenario called the ‘Strange Situation’. In this experiment, the baby is introduced to a sequence of events and the response of the baby is scored by a panel of observers. The series of events which we will consider in this report, called the *Strange Situation scenario*, are,

1. Baby carried in to room by Mother.
2. Baby put down by Mother.
3. Stranger enters room, offers baby a toy and the Mother leaves.
4. Mother absent, Stranger present. Stranger lets Baby play if okay, tries to engage Baby if Baby looks distressed.
5. Mother returns.
6. Mother and Stranger leave.
7. Mother stays out, Stranger returns.

8. Mother returns, tries to pick up Baby.

The response of the baby is measured in a number of ways, for example by counting the number of times it approaches the mother (proximity and contact seeking), it looks towards the mother (distant interaction), it ignores the mother (avoidance), it remains focussed on the mother (contact maintaining with Mother) and angry resistance upon reunion (resistance). As a result of these measurements, a classification of the baby's attachment style is made into three broad categories, Dismissive, Anxious or Secure. These categories may be further subdivided.

The key question is given a baby and a mother, how does the baby's attachment style develop over the first eighteen months of its life. It would also be valuable to be able to make an objective classification of the baby's type based on the Strange Situation data. In the next section the statistical tests that can be used to help categorise a baby are discussed. Then in Section 3 some plausible dynamical models for the development of the attachment style are presented. Finally in Section 4 we discuss briefly the merits of the different approaches.

2 The Classification problem

Data exists as to how an archetypal baby in each attachment style behaves in the Strange Situation scenario. An example of such data taken from the second, third, fifth and eighth episodes in the Strange Situation is given in Table 1.

Each of the numbers indicates a mean frequency of the number of times the behaviour pattern is observed for each type. Note that some of the observations discriminate between the different attachment classifications more readily than others. For example, all babies have a similar frequency count for their distant interaction when put down by the mother, but the different attachment types respond significantly differently in their contact maintaining behaviour with the mother when the mother first returns. Note also that the frequency of some events is much greater for some of the types of observations than others. For example, the resistance count when the stranger enters is typically much lower than the avoidance count when the mother returns for the second time.

We define the matrices $\mathbf{D}, \mathbf{S}$ and $\mathbf{A}$ as the sets of values that give the typical behaviour for each of the attachment styles, Dismissive, Secure and Anxious respectively. The matrix of mean values, $\boldsymbol{\mu}$ is constructed, where

$$\mu_{ij} = \frac{D_{ij} + S_{ij} + A_{ij}}{3},$$

and then the matrix for the root mean square for each measurement, $\boldsymbol{\sigma}$, calculated from

$$\sigma_{ij}^2 = \frac{1}{2}\left((D_{ij} - \mu_{ij})^2 + (S_{ij} - \mu_{ij})^2 + (A_{ij} - \mu_{ij})^2\right).$$

		Baby put down(2)	Stranger enters(3)	Mother returns (5)	Mother returns (8)
Proximity and contact seeking	D	1.5	1.8	1.8	2.3
	S	2.2	4.9	3.3	4.2
	A	2.2	3.1	3.6	3.9
Contact maintaining with mother	D	1.2	1.4	1.0	2.0
	S	1.8	1.8	2.2	4.2
	A	2.4	2.9	4.0	4.4
Resistance	D	1.0	1.0	1.1	2.5
	S	1.1	1.0	3.1	3.5
	A	1.7	1.3	3.3	4.0
Avoidance	D			4.9	5.4
	S			4.8	4.3
	A			2.0	2.6
Distant Interaction	D	2.8	1.4	2.7	2.4
	S	2.8	2.0	2.5	1.7
	A	2.5	1.7	1.3	1.1

Table 1: Table of responses of archetypes in Strange Situation episodes 2, 3, 5 and 8. (D - Dismissive, S -Secure, A-Anxious).

The observations of the baby that is being tested are then made, and the frequency counts placed in a matrix $\mathbf{B}$.

How closely the test baby resembles each of the archetypes may now be measured by evaluating the distance between $\mathbf{B}$ and each of $\mathbf{D}, \mathbf{S}$ and $\mathbf{A}$ by calculating

$$
\begin{aligned}
||\mathbf{D}-\mathbf{B}|| &= \Delta D = \sum_{i,j} \frac{(D_{ij}-B_{ij})^2}{\sigma_{ij}^2}, \\
||\mathbf{S}-\mathbf{B}|| &= \Delta S = \sum_{i,j} \frac{(S_{ij}-B_{ij})^2}{\sigma_{ij}^2}, \\
||\mathbf{A}-\mathbf{B}|| &= \Delta A = \sum_{i,j} \frac{(A_{ij}-B_{ij})^2}{\sigma_{ij}^2}.
\end{aligned}
$$

Weighting each difference by σ_{ij}^2 takes account of the fact that some elements of the archetype matrices $\mathbf{D}, \mathbf{S}$ and $\mathbf{A}$ are better at discriminating between the different types than others and scales each element to the same relative magnitude.

The outcome of this is three numbers, $\Delta D, \Delta S$ and ΔA that measure how different the baby is from each of the archetypes: the larger the value the less the test baby resembles the relevant archetype. This information can be represented graphically by plotting an equilateral triangle of side length λ where each apex represents one of the three states $\mathbf{D}, \mathbf{S}$ or $\mathbf{A}$ and a point in the triangle indicates approximately the relative distance of the test baby from each of these states.

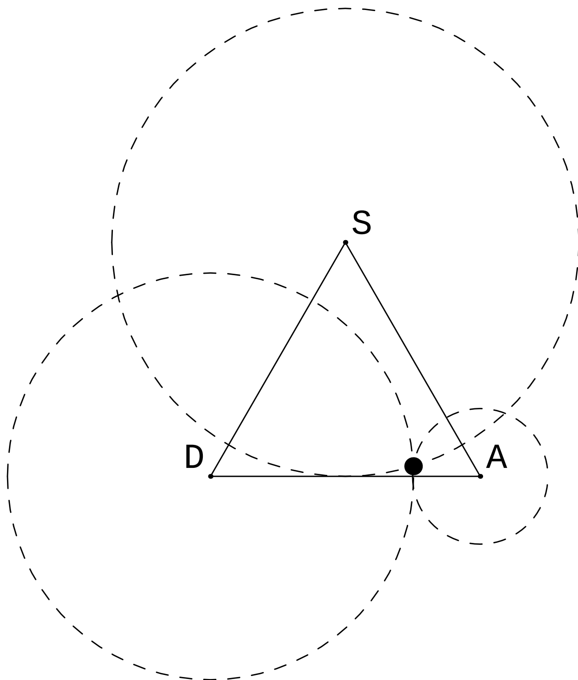

Figure 1: Attachment triangle showing the position of a baby with $\Delta D = 0.3, \Delta S = 0.4$ and $\Delta A = 0.1$.

An example is shown in Figure 1 where the three archetypes are represented by the apexes at the coordinates $\mathbf{A} = (A_x, A_y) = (\lambda/2, -\lambda/(2\sqrt{3}))$, $\mathbf{D} = (D_x, D_y) = (-\lambda/2, -\lambda/(2\sqrt{3}))$ and $\mathbf{S} = (S_x, S_y) = (0, \lambda/\sqrt{3})$. The origin is at the centre of the triangle and is equidistant from each of the three archetypes. The coordinates (X, Y) of a test baby with values $\Delta D, \Delta A$ and ΔS are given by the solution of the three equations

$$
\begin{aligned}
(X - A_x)^2 + (Y - A_y)^2 &= \mathrm{Min}\{r^2 \Delta A^2, 3\lambda/4\} \\
(X - D_x)^2 + (Y - D_y)^2 &= \mathrm{Min}\{r^2 \Delta D^2, 3\lambda/4\} \\
(X - S_x)^2 + (Y - S_y)^2 &= \mathrm{Min}\{r^2 \Delta S^2, 3\lambda/4\}.
\end{aligned}
$$

representing the locus of the three circles shown in Figure 1.

This single point can only indicate relative distance from each of the states and not absolute distances so, for example, a baby with overall values $\Delta D = 1, \Delta S = 1$ and $\Delta A = 2$ would be represented by the same point as a baby with values $\Delta D = 10, \Delta S = 10$ and $\Delta A = 20$. However, in the former case it is more likely that the baby is represented well by the model of three attachment states. It would therefore be advisable to not only plot a triangle to indicate the relative position of the test baby, but also record the values of $\Delta D, \Delta S$ and ΔA.

The matrix $\boldsymbol{\sigma}$ is one way of verifying which elements of the matrices are actually most important: a subset of the 18 values given in Table 1 is probably sufficient.

More sophistication can be introduced in the analysis if more information is available. For example, if the standard deviation for each of the measurements for the Detached, Secure and Anxious categories are available then, under the assumption that the measurements are uncorrelated, a chi-squared test can be performed. As before matrices $\mathbf{D}, \mathbf{S}$ and $\mathbf{A}$ are defined but in addition we have three matrices $\boldsymbol{\sigma}_D$, $\boldsymbol{\sigma}_S$ and $\boldsymbol{\sigma}_A$ giving the standard deviations in each measurement. Given now a matrix $\mathbf{B}$ for the baby being tested, the three quantities χ^2_D, χ^2_S and χ^2_A are constructed where

$$\chi^2_D = \sum_{i,j} \frac{(D_{ij} - B_{ij})^2}{\sigma^2_{Dij}} \quad (1)$$

$$\chi^2_S = \sum_{i,j} \frac{(S_{ij} - B_{ij})^2}{\sigma^2_{Sij}} \quad (2)$$

$$\chi^2_A = \sum_{i,j} \frac{(A_{ij} - B_{ij})^2}{\sigma^2_{Aij}}. \quad (3)$$

Dividing by the standard deviation has the effect of scaling the contribution of each term according to how good a measure of the state it is based on the width of the distribution of measured values. Now it is the chi-squared value in each case that gives a measure of how closely the test baby resembles each of the states, where again the smaller the value the closer the baby is to the archetype. The results can be presented graphically in the same manner as for the first test, with once again a proviso that the smallest chi-squared value would give an indication of how well the test baby fits the attachment model. An indication of the relative importance of each particular measurement in determining the overall value can be obtained by plotting a histogram of the contributions for each term to the overall value of χ^2_D, χ^2_S and χ^2_A.

If all the data, case by case, that went in to constructing Table 1 was available, then it would be possible to do a more detailed analysis. Firstly, a test to establish how well the data agreed with the notion of three attachment styles (or three main attachment styles plus various sub-categories) could be performed. Secondly, the complete dataset would enable the full error matrix to be computed. This would tell one how strongly correlated the different measurements are. For example, if one entry goes up, does another entry also always go up? With the inverse of the error matrix the full chi-squared test can then be computed that takes the correlations into account. This would result in a more accurate measure for how far any test baby was from each of the attachment categories.

The full dataset would also allow us to analyse the classification problem using traditional classification methods. We would need a supervised classification method such as linear or quadratic discrimination where the classes or categories are defined using a *training set* of babies representing each attachment style (obtained from the full dataset). We can think of the matrices **D**, **S** and **A** as $k \times 1$ dimensional vectors $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \ldots, \zeta_k)$ with a general vector $\mathbf{X}$ to be classified into one of three classes ($C_1 = S, C_2 = D$ and $C_3 = A$). The mean response score for class i is given by

$$\boldsymbol{\mu}_i = \frac{1}{n_i} \sum_{\boldsymbol{\zeta} \in C_i} \boldsymbol{\zeta} \tag{4}$$

where the sum is over all n_i vectors in class C_i. In group i the random vector $\boldsymbol{\zeta}$ is assumed to have a multivariate normal density with mean $\boldsymbol{\mu}_i$ and variance-covariance matrix $\boldsymbol{\Sigma}_i$. The methods are based on maximum likelihood. For *linear discrimination* the variance-covariance matrices are assumed to be the same, that is $\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \boldsymbol{\Sigma}_3$. Then $\boldsymbol{\Sigma}$ can be estimated as

$$\hat{\boldsymbol{\Sigma}} = \sum_{i=1}^{3} \left(\sum_{\boldsymbol{\zeta} \in C_i} \boldsymbol{\zeta}\boldsymbol{\zeta}^T - n_i \boldsymbol{\mu}_i \boldsymbol{\mu}_i^T \right) / \sum_{i=1}^{3} n_i. \tag{5}$$

Distances in the k-dimensional space are measured taking into account the variance-covariance matrix. The squared Mahalanobis distance between vectors $\boldsymbol{\zeta}$ and the i^{th} category mean $\boldsymbol{\mu}_i$ is [3]

$$(\boldsymbol{\zeta} - \boldsymbol{\mu}_i)^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{\zeta} - \boldsymbol{\mu}_i). \tag{6}$$

We classify each baby by the nearest mean as measured by the above distance. This rule simplifies to give a set of linear thresholds. In the case where there are three categories the rule will be based on three functions h_{12}, h_{13} and h_{23}, where

$$h_{ij}(\mathbf{X}) = \left(\boldsymbol{\mu}_i - \boldsymbol{\mu}_j\right)^T \boldsymbol{\Sigma}^{-1} \left(\mathbf{X} - \frac{(\boldsymbol{\mu}_i + \boldsymbol{\mu}_j)}{2}\right), \quad i, j = 1, 2, 3, \ i \neq j. \tag{7}$$

Noting that $h_{ij}(\mathbf{X}) = -h_{ji}(\mathbf{X})$, we classify the baby with vector $\mathbf{X}$ to

$$\begin{aligned}
&C_1 \text{ if } \ h_{12}(\mathbf{X}) > 0 \text{ and } h_{13}(\mathbf{X}) > 0, \\
&C_2 \text{ if } \ h_{21}(\mathbf{X}) > 0 \text{ and } h_{23}(\mathbf{X}) > 0 \\
&\text{and} \\
&C_3 \text{ if } \ h_{31}(\mathbf{X}) > 0 \text{ and } h_{32}(\mathbf{X}) > 0.
\end{aligned}$$

If the three group variance-covariance matrices are not assumed to be equal, the procedure results in quadratic discrimination, giving a set of curved thresholds. Other methods of supervised classification include logistic discrimination and machine learning methods such as neural network discrimination, tree-based classification and nearest neighbour classification.

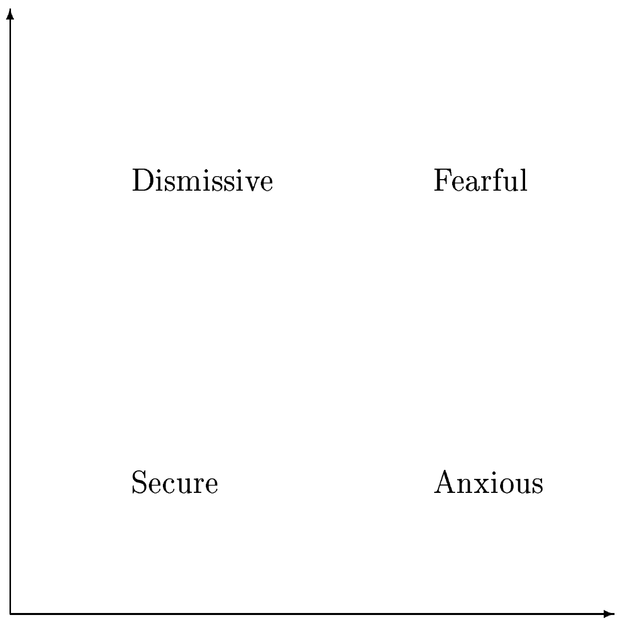

Figure 2: The four attachment styles

3 Dynamical models

In a four category model of attachment, each individual's attachment style can be categorised as one of Secure, Anxious, Dismissive, and Fearful and can be represented by the two-dimensional diagram shown in Figure 2. A fifth attachment style, Disorganised, exists where an individual's attachment style wanders between each of Secure, Anxious, Dismissive and Fearful.

The process by which an individual develops a particular attachment style is complex and difficult to model. The attachment process evolves in time and two key questions arise : is there a small number of measurable quantities that capture the time evolution of this process? Is good quality data available to help validate any model that is derived?

Below we suggest two possible models and discuss their advantages and disadvantages.

Model 1

In this first model we adopt the view that, since we wish to have as an outcome a position on the attachment plane shown in Figure 2, the dependent variables that evolve with time should be direct measures of one's feelings about oneself and one's feelings about others. Specifically, we suppose that C_1 is a measure of how insecure an individual is with themselves and C_2 is a measure of how insecure an individual is with regard to others. We suppose that $C_1 = C_2 = 0$

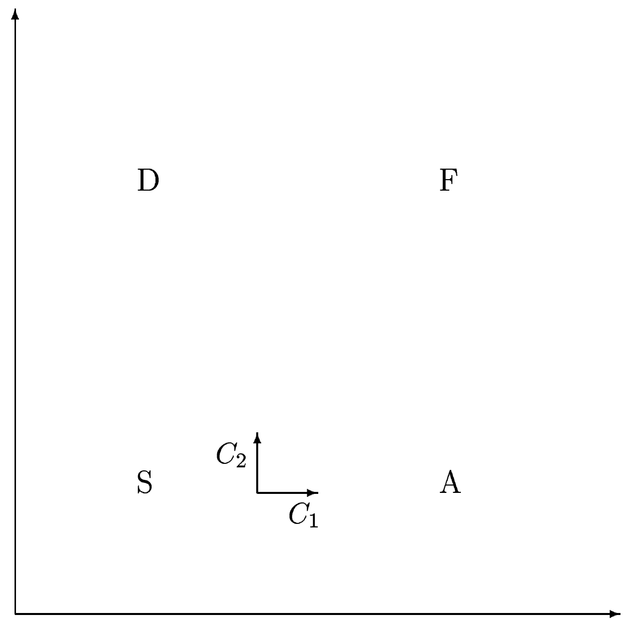

Figure 3: The four attachment styles related to C_1 and C_2

represents an individual's inherent attachment style in the absence of external factors, so $C_1 = C_2 = 0$ will occur at different points on Figure 2 depending on an individual's personality. Note that an alternative definition (which we don't pursue here) would be to fix the origin at $C_1 = C_2 = 0$ for all individuals. An example showing how C_1 and C_2 relate to the attachment plane for an individual who is inherently quite secure but with anxious tendencies is shown in Figure 3.

One model of how C_1 and C_2 evolve with time is

$$\begin{aligned} \frac{dC_1}{dt} &= -aC_1 + f(t) + bC_1C_2 \\ \frac{dC_2}{dt} &= -dC_2 + g(t) + kC_1C_2. \end{aligned} \tag{8}$$

The first term on the right hand side of each equation describes the natural tendency, in the absence of any external factors, for C_1 and C_2 to return to their inherent equilibrium value $C_1 = C_2 = 0$. Note that we take a and d to be strictly positive; if they were negative then even in the absence of any interaction C_1 and C_2 would tend to diverge from $C_1 = C_2 = 0$. Here, $f(t)$ and $g(t)$ represent changes to the environment that may be constant or time-varying, or even contain sudden jumps such as might occur if there was a sudden change in the individual's economic circumstances. When $f(t)$ is positive it has a tendency to increase C_1, and similarly when $g(t)$ is positive it has a tendency to increase C_2. The final terms on the right hand side represent the coupling between C_1 and C_2: if an adverse event occurs that reduces one's confidence in others thereby increasing C_2, then one's response will be greater the less secure one is with oneself. Similarly, events that tend to reduce one's self-confidence have a greater effect the less secure one is with others. The signs of b and k determine whether the response is to increase or decrease the values of C_1 and C_2.

In the simplest case, $f(t) = g(t) = 0$. If either b or k is zero then $C_1 = C_2 = 0$ is the only equilibrium point and it is a stable node. When both b and k are non-zero then the system of equations (8) has two steady-state solutions, $C_1 = C_2 = 0$ and $C_1 = d/k$ and $C_2 = a/b$. A linear stability analysis shows that the former is always stable and that the latter is a saddle, regardless of the signs of the interaction terms. Note that whilst the origin is always feasible it is possible that the other equilibrium point is infeasible for certain parameter values. For the purposes of the discussion below, we assume that the equilibrium points are all feasible.

We now suppose that the environmental factors are constant but non-zero and let $f(t) = \alpha$ and $g(t) = \beta$. When these are positive they correspond to constant effects that tend to increase one's self-insecurity and insecurity about others, respectively. Provided α and β are not too large, the phase plane showing the time evolution of C_1 and C_2 is qualitatively similar to that with no environmental input, the prime difference being that the equilibrium values are shifted. This is shown below. First, in order to simplify the sketching of the phase plane, we remove one of the free parameters by rescaling time. Specifically, we let $\tau = bt$, so that (8) becomes

$$\frac{dC_1}{d\tau} = u - AC_1 + C_1C_2 \qquad (9)$$

$$\frac{dC_2}{d\tau} = r(v - DC_2 + C_1C_2), \qquad (10)$$

where $r = \frac{k}{b}, u = \frac{\alpha}{b}, v = \frac{\beta}{k}, A = \frac{a}{b}$ and $D = \frac{d}{k}$. The nullclines are thus

$$C_2 = A - \frac{u}{C_1} \text{ and } C_2 = \frac{-v}{C_1 - D}, \qquad (11)$$

and are sketched in Figure 4.

Equilibrium solutions, $(C_1, C_2) = (X, Y)$ occur at the intersection of the null-clines. Solving for X using (11) we obtain

$$AX^2 + (v - u - AD)X + uD = 0.$$

The X components of the possible steady states are thus

$$X_+ = \frac{AD + u - v}{2A} + \frac{1}{2A}\Delta,\ X_- = \frac{AD + u - v}{2A} - \frac{1}{2A}\Delta,$$

where $\Delta = \sqrt{(AD + u - v)^2 - 4uAD}$. Hence the steady state solutions are

$$S_+ = \left(X_+, \frac{v - u + AX_+}{D}\right) \text{ and } S_- = \left(X_-, \frac{v - u + AX_-}{D}\right).$$

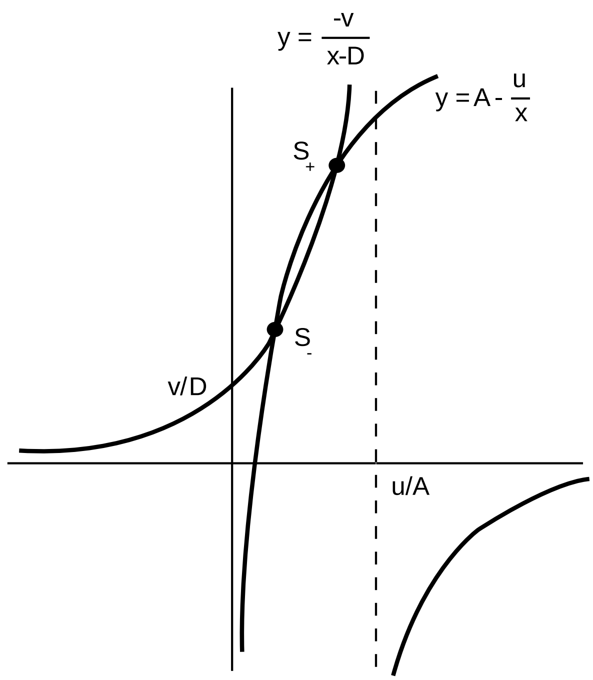

Figure 4: **Nullclines for model**

In order to find the stability of these steady states, the Jacobian J is computed where

$$\mathbf{J} = \begin{pmatrix} Y - A & X \\ rY & r(X - D) \end{pmatrix}.$$

The signs of the eigenvalues of $\mathbf{J}$ give the stability of the steady state. In the specific case where both steady-states are positive, we have that $X < D$ and $Y < A$. Then

$$\text{trace}\ (\mathbf{J}) = Y - A + r(X - D) < 0$$

and

$$\begin{aligned} \det\ (\mathbf{J}) &= r(X - D)(Y - A) - rXY \\ &= r(DA + u - v - 2AX) \\ &\quad < 0 \text{ for } X = X_+ \\ &\quad > 0 \text{ for } X = X_-. \end{aligned}$$

Hence S_- is a stable node, whereas S_+ is a saddle (see Figure 5). Note that the stable steady-state solution S_- has shifted away from the origin towards positive values of C_1 and C_2. In other words, if one is subjected to small constant negative influences that tend to increase your insecurity either in yourself or in others, then this will be reflected in a less secure attachment state.

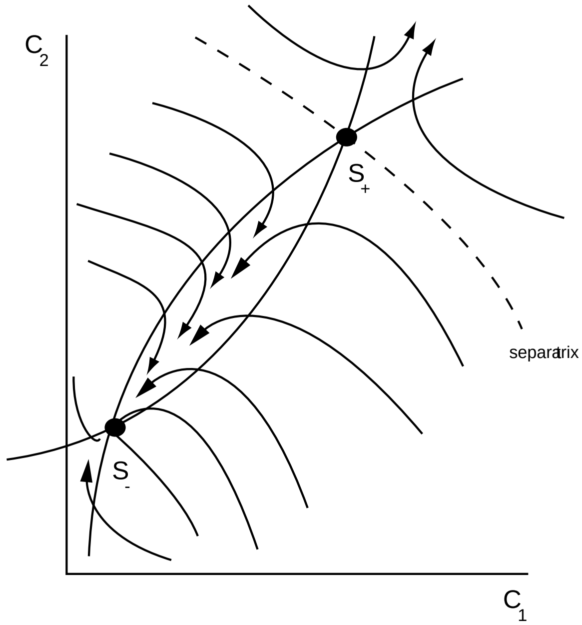

Figure 5: **Phase plane for model**

As the values of α and β, are increased to

$$\beta = \frac{1}{b}(ad + \alpha k - \sqrt{4\alpha adk}),$$

then, ignoring whether they are still feasible, the two steady-state solutions vanish in a saddle-node bifurcation. The constant negative influence has had the effect of removing any possibility of settling to a stable attachment state. We note that as α and β are further increased to

$$\beta = \frac{1}{b}(ad + \alpha k + \sqrt{4\alpha adk}),$$

then the two steady state solutions reappear again in a reverse saddle node bifurcation again ignoring questions of feasibility.

The particular structure of the phase plane as a function of the parameters is highly dependent on the choice of the form of the nonlinear interaction terms. Here we took terms of the form C_1C_2. More realistic choices would take account of the fact that C_1 and C_2 both positive presumably does not have the same effect as values of C_1 and C_2 that have the same magnitude but both negative.

Model 2

Mathematical models of marriage have been constructed which model relationships as a set of discrete events that change the state of ‘happiness’ of an individual [2]. An individual’s happiness increases if a positive comment is made, but decreases if a negative comment is made. Typically the functions modelling the response to a comment were taken to be asymmetric in that a negative comment had a stronger impact than a positive comment.

We can seek to take a similar approach here. Once again a central issue is what to take as natural variables to model the attachment state. In what follows, we choose u to be a measure of the self-insecurity of an individual and u is restricted to lie between zero and one, where zero and one indicate extremes of security and insecurity respectively. A second variable v is a measure of how insecure an individual is about receiving support from others. This second variable also lies between zero and one. We assume that if nothing happens then u and v do not change. In an 'event' u and v change. These events can be of three types: type one only changes u, type two only changes v, and type three changes both u and v. A type one event might be where a baby attempts to do something for themselves and either fails or succeeds. A type two event might be where a baby would like a particular response from the mother but does not receive it. A type three event could be viewed as a mixture of one and two. A simple model would be

$$\begin{aligned} u^{(n+1)} &= u^{(n)} + E_1(u^{(n)}, v^{(n)}, n) \\ v^{(n+1)} &= v^{(n)} + E_2(u^{(n)}, v^{(n)}, n), \end{aligned} \tag{12}$$

where $u^{(0)} = a$ and $v^{(0)} = b$. The choice of a and b determine the initial state of the baby. The key issue is in the choice of the functions E_1 and E_2. These need to be such that the left hand side of equations (12) are mappings on the unit square to ensure that u and v remain bounded between zero and one. In making a reasonable guess at the functional form for E_1 we assume that

1. If you are very secure in yourself (low u value), then you are relatively insensitive to events.

2. If you are very insecure in yourself (high u value), then you are also at an extreme of behaviour and also relatively insensitive to events.

3. Being secure in the knowledge that others will help you (low v value) also makes you relatively insensitive.

4. Being very insecure in the expectation that others will help you (high v value) makes you relatively insensitive.

5. One's responsiveness to an event in terms of a change in u or a change in v decreases with time.

With analogous assumptions for E_2, one choice is

$$\begin{aligned} E_1 &= \alpha(n) \sin^2 \pi u \sin^2 \pi v \exp(-n/\gamma), \\ E_2 &= \beta(n) \sin^2 \pi u \sin^2 \pi v \exp(-n/\gamma), \end{aligned} \tag{13}$$

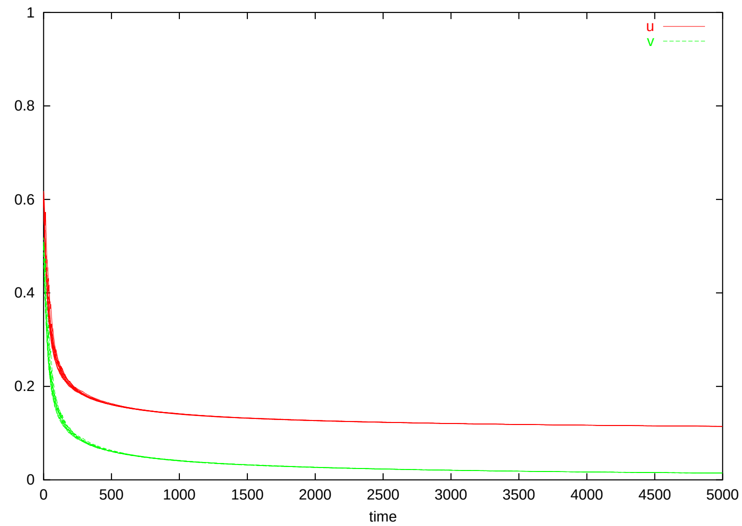

Figure 6: Time series for Model 2 in the case $u^{(0)} = 0.6, v^{(0)} = 0.5, \gamma = 5000$. For 90% of the time $\alpha = \beta = -0.01$ and for 10% of the time $\alpha = \beta = 0.01$. Ten separate runs are superimposed.

with $\alpha(n)$ and $\beta(n) \in [-0.439, 0.439]$. The $\sin^2$ terms are essentially one hump maps of the unit interval. There are many similar choices that could be made, including piecewise linear functions that may make analysis easier but are likely not to significantly change the qualitative behaviour of the model. The exponential term has the effect of decreasing the value of the interaction term over time. Crucially this means that the system will always eventually settle to a fixed attachment style. The parameter γ gives a measure of roughly how long it will take for this settled attachment style to be reached. The parameters $\alpha(n)$ and $\beta(n)$ could be constant, but more realistically would vary at each step depending on whether the response received by the baby from the mother had the effect of increasing or decreasing their security values. For example, one might imagine that a mother could provide a good response for the baby 90% of the time, but a poor response 10% of the time.

The dynamics of the model are extremely rich. Two examples are shown in Figures 6 and 7. These show the time evolution of u and v for different parameter values. In each of these cases the baby starts out being slightly anxious about themselves, $u^{(0)} = 0.6$, and to have a neutral view as to the behaviour of others, $v^{(0)} = 0.5$. The time scale γ has been set to 5000. We have only allowed for type three events, and so the behaviour of the two variables mimic each other. In both cases shown, the baby receives good responses 90% of the time and bad responses 10% of the time, so 90% of the time α and β are negative giving a tendency for u and v to decrease, whilst 10% of the time α and β are positive giving a tendency for u and v to increase. In Figure 6 both good and bad events

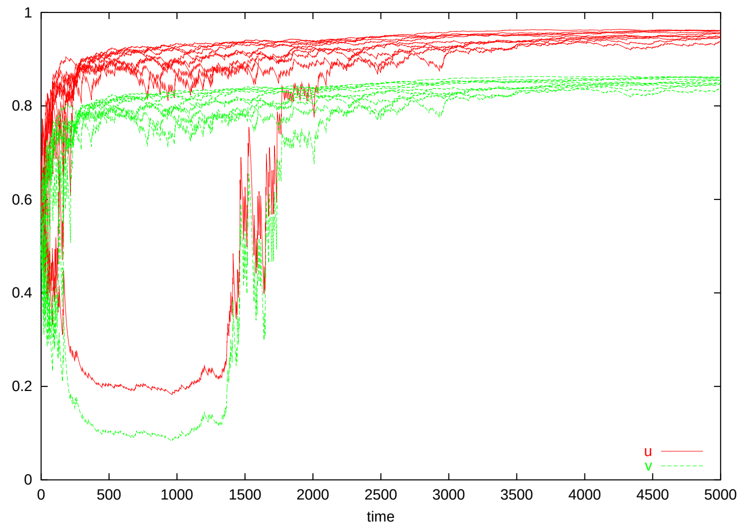

Figure 7: Time series for Model 2 in the case $u^{(0)} = 0.6, v^{(0)} = 0.5, \gamma = 5000$. For 90% of the time $\alpha = \beta = -0.01$ and for 10% of the time $\alpha = \beta = 0.1$. Ten separate runs are superimposed.

are possible and so α and β either take the values -0.01 or 0.01. In this case, one sees from the time series that the good events outweigh the bad events and the ultimate result is a secure baby. Since the selection of whether an event is good or bad is made randomly, ten different time series have been superimposed to show that regardless of the order of the good and bad events, the final outcome is the same.

In contrast, in Figure 7 bad events are more heavily weighted than good events, so that α and β either take the values -0.01 or 0.1. Again ten different runs are superimposed to indicate how dependent the final outcome is on the order of the events. Here, although only one in ten events are bad, because one bad event has ten times the effect of one good event the net outcome is an insecure baby.

4 Discussion

Any mathematical model of a process, be it physical, chemical, biological or psychological only has use as a predictive tool if it can be validated against real data. The prime difficulty in deriving a model for the development of attachment relationships is trying to establish measurable quantities that are observable in an experimental situation. The ordinary differential equation example in Model 1 highlights the difficulties: if this is to provide real insight in to the development of attachment styles, then key questions are: how to measure C_1 and C_2? How do

C_1 and C_2 depend on each other? How can experimental observations be related to the values for the various parameters in the model? Model 2 suffers from many of the same difficulties, but there is perhaps more hope that one could establish the qualitative functional form of the interaction terms, E_1 and E_2. From video recording a mother and baby in their natural environment, it may be possible to establish at least the statistical pattern of how often a mother responds well to a baby's needs and how often not; how often a mother encourages the baby and how often not. One further key question here is what constitutes an 'event'?

In both Models 1 and 2, it is assumed that the parent's interaction with the baby is not coupled with the way the baby behaves. This is clearly a simplification and can be removed at the expense of making the models more complicated. Whichever model is investigated, the robustness of the model to any assumptions made would need to be investigated.

In terms of analysing data from the Strange Situation, statistics has much to offer. Given only information of the type given in Table 1 the measure suggested in equations 1 are sensible. Given standard deviation information for each of the numbers in Table 1 it would be possible to do a chi-squared test under the assumption that the different measurements for each archetype are uncorrelated. If all the data, case by case, that went in to making up Table 1 was available, then further statistical tests could be carried out, as outlined in Section 2.

Just as statistical tools can be used, and perhaps already have been used, to analyse the Strange Situation baby, they could be used to measure an adult attachment style. In order to do this, whatever experiment, test or questionnaire was used would have to be carefully designed to allow the attachment style to be deduced. Values for the observations/answers for each of the archetypes would have to be found, for example, by running the experiment on individuals whose attachment style was well-established by independent means. The kind of measures discussed in Section 2 and above in this section could then be used.

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