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< http://www.maths-in-medicine.org/uk/2010/swallowing/ >

Mathematical modelling of the normal swallow

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1 Introduction

Dysphagia refers to a difficulty with eating drinking or swallowing with an estimated 40% of adults presenting with dysphagia following stroke. This figure does not include populations who develop dysphagia due to other disease processes such as Parkinsons disease and dementia. The risks of dysphagia include; developing aspiration pneumonia, malnutrition and dehydration. Management of dysphagia includes provision of modified foods and drinks to attempt to address this problem. This management is based on the assumption that these foods/liquids have the best rheological properties for people with dysphagia, based on current knowledge of the normal swallowing process. However there is currently no complete mathematical model of the whole of the dynamic swallow process. A model which incorporates each of the stages of the normal swallow and considers the transitions and relationships between the stages would not only inform current knowledge, but also allow future investigation of specific types of swallowing disorders and help us look more accurately at the impact of modified foods and liquids on the swallowing process.

Mathematical modelling is now possible due to an increase in the information available such as, impact of bolus size, timing of parts of the swallow and measurement of pressures in specific parts of the swallow, such as the pharyngeal stage and models of fluid flow at the point of ejection of the oral cavity. Whilst these all give relevant information this is has not been utilised to produce a complete mathematical model of either a swallow stage or the whole swallow process.

The problem presented at the Study Group considered the swallowing process which we could either look at as a whole or a series of stages. The normal swallow is a dynamic process. It is normally divided up into 4 stages:

Oral preparatory stage. Food and drink is taken into the mouth and controlled within the oral cavity by the lingual, buccal, palatal and lip musculature (see Figure 1). In this stage the food is manipulated and chewed, and mixed with saliva to form a bolus (material

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ready to swallow). Food and liquid are prevented from falling backwards via the posterior sealing of the tongue and the soft palate. The cheek muscle (buccinator) is tensed to prevent lateral spill and the lips are closed so no material escapes anterioraly. The bolus is held between the tongue and the hard palate with the intrinsic muscles of the tongue altering tongue shape to accommodate the bolus size and consistency. The velopharyngeal port is open and respiration continues. There is a great deal of individual variation in the length and pattern within this stage which requires intact muscles of mastication.

Oral stage. The bolus is propelled posterioraly in the oral cavity by the tongue which forms a central groove and performs a stripping action against the hard palate, pushing the bolus backwards. As the bolus moves the posterior tongue moves down to allow the bolus to move backwards and the soft palate elevates (preventing food being regurgitated through the nose), expanding the pharyngeal space for the bolus. When the bolus reaches the posterior tongue at the faucal arches the next stage of the swallow begins as the bolus moves to the oropharynx.

Pharyngeal stage. Respiration and swallowing have to be coordinated in this reflex swallow stage. The bolus is moved posteriorly by the tongue into the oropharynx. The hyoid bone begins to elevate and then move anterioraly, moving the larynx up and forwards under the root of the tongue, expanding the lower pharynx and causing a decrease in pressure in front of the bolus. This decrease in pressure and the piston action of the tongue drives the bolus through the pharynx. The pharyngeal constrictors contract and the pharynx rises then falls with contraction to make a peristaltic wave also moving the bolus down. Within the larynx, closure to protect the trachea begins with the superior movement of the larynx. The true vocal folds close followed by the false vocal folds and then retroversion of the epiglottis takes place which diverts the bolus laterally towards the pyriform sinuses. It is the true vocal fold adduction which provides the main protective mechanism to prevent aspiration during the swallow. During this closure is a period of apnoea which may last from 0.3-2.5 seconds. As the larynx raises and tilts, this supports the opening of the cricopharyngeal sphincter which is the first sphincter opening into the oesophagus. The movement of the larynx pulls the cricoid lamina away from the posterior pharyngeal wall resulting in a stretching effect. This in combination with the relaxation of the upper oesophageal sphincter creates space and a pressure differential for the bolus to be moved into the oesophageus. The larynx and hyoid bone return to their resting position and respiration recommences.

Oesophageal stage. This is also a reflex stage and is where the food/liquid is moved down to the lower oesophageal sphincter by peristalsis. The normal swallow however is a dynamic event with a number of processes occurring at the same time. For example when chewing a large mouthful, some parts of this are swallowed before others so more than one stage occurs at the same time. This is however dependent on coordinated intact musculature and neurological innervation.

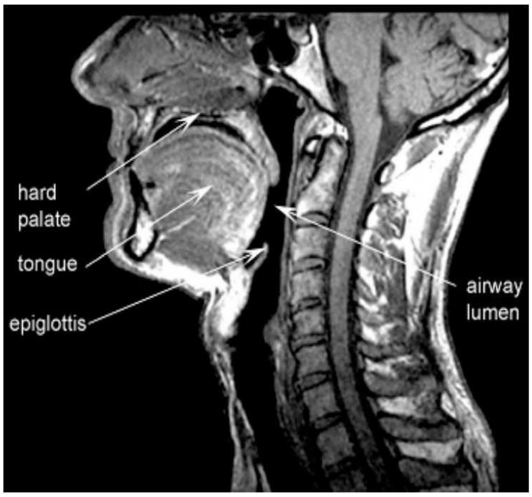


Figure 1: *A magnetic resonance image (MRI) of the structures involved in the swallowing process. The anterior oral cavity is where the pre-bolus formation takes place. The tongue movement presses the bolus against the hard palate and propels it backwards. The hyoid bone moves upwards then forward, raising and tilting the larynx and closing the laryngeal space which supports the opening of the upper oesophageal sphincter. In the oesophagus, peristalsis movement which is already known and is not considered in this report. Image taken from [8].*

2 Mathematical modelling

Although the normal swallow is divided into four stages we only focus our attention on the first three stages. The last Oesophageal stage, where the bolus movement is controlled by peristalsis, is quite well understood and there are plenty of models available in literature to describe it [4, 9].

We will now start building on a mathematical model for the the swallowing process in two dimensions. Note that this is a considerable simplification since the imaging data suggests that swallowing is a strongly three dimensional process. However, the two dimensional problem does seem to be a good starting point as far as we are concerned.

The main structural elements involved in the swallowing process which will be considered here are the tongue and pharynx. For simplicity, we assume the tongue and the pharynx to be one straight unit. Figure 2 shows a schematic of the set-up. The hard palate (upper wall) is rigid, but the tongue is elastic and flexible. To simplify the problem further, we either prescribe the tongue movement or the applied pressure resulting in tongue movement (see Figure 2). The mechanisms driving the liquid or bolus are the tongue induced drag of fluid (by stretching), inertia, gravity and peristalsis.

The fluid mechanics of the liquid/bolus is described using the well-known Navier-Stokes

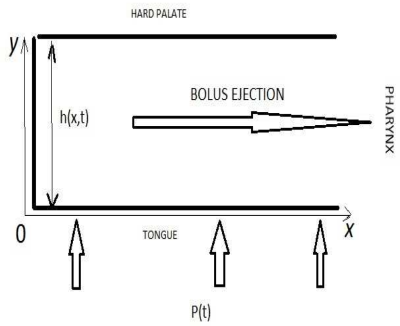


Figure 2: *Diagram of a simplied model for oral stage bolus ejection. The movement of the tongue (bottom moving wall) is prescribed by either giving the height as a function of time and space, $h(x,t)$, or by specifying the pressure applied by the tongue as a function of time and/or space, i.e. $P(t)$ or $P(x;t)$ (the pressure applied by the tongue can depend on the x coordinate if the tongue is considered an elastic wall).*

equations:

$$\rho(u_t + uu_x + wu_y) = -P_x + \mu(u_{xx} + u_{yy}) + \rho g, \tag{1}$$

$$\rho(w_t + uw_x + ww_y) = -P_y + \mu(w_{xx} + w_{yy}), \tag{2}$$

$$u_x + w_y = 0, \tag{3}$$

where (u, w) are the fluid velocities in the x, y directions, respectively, P is the fluid pressure, μ is the fluid viscosity and ρ is the fluid density. The boundary conditions prescribed are:

$$u = w = 0, \text{ at } y = 0 \text{ and } u = f(x,t), \; w = h_t, \text{ at } y = h(x,t),$$

where $h(x,t)$ is prescribed (representing tongue movement) and $f(x,t)$ describes the stretching of the tongue. Note that the function $h(x,t)$ can be obtained from analysing the imaging data.

We non-dimensionalise the above equations by

$$x = Lx', \quad (y,h) = H(y',h'), \quad u = Uu', \quad w = Ww',$$
$$p = P_0 p', \quad t = Tt', \quad f = f_0 f', \tag{4}$$

where the primed variables are dimensionless and the characteristic quantities for the length, velocities, pressure and time are given by L, H, U, W, P_0 and T, respectively, will be determined by appropriate balances of the relevant physical mechanisms described above. Substituting Eq. (4) into equations (1)-(3) and the boundary conditions (equation (2)), we obtain

$$\rho\left(\frac{U}{T}u'_{t'}+\frac{U^2}{L}u'u'_{x'}+\frac{UW}{H}w'u'_{y'}\right) = -\frac{P_0}{L}P'_{x'}+\mu\left(\frac{U}{L^2}u'_{x'x'}+\frac{U}{H^2}u'_{y'y'}\right)+\rho g, \quad (5)$$

$$\rho\left(\frac{W}{T}w'_{t'}+\frac{UW}{L}u'w'_{x'}+\frac{W^2}{H}w'w'_{y'}\right) = -\frac{P_0}{H}p'_{y'}+\mu\left(\frac{W}{L^2}w'_{x'x'}+\frac{W}{H^2}w'_{y'y'}\right), \quad (6)$$

$$\frac{U}{L}u'_{x'}+\frac{W}{H}w'_{y'} = 0, \quad (7)$$

with

$$Uu' = f_0 f(x',t'), \quad Ww' = \frac{H}{T}h'_{t'}, \quad \text{at} \quad y'=0. \quad (8)$$

and

$$u' = w' = 0, \quad \text{at} \quad y' = h'(x',t'). \quad (9)$$

From Eq. (7), one obtains $W = \frac{UH}{L}$. Using this in the last boundary condition in Eq. (8) and (9), gives $T = \frac{L}{U}$.

Multiplying the first two equations by $\frac{L}{\rho U^2}$ and using the scalings for W, T, gives

$$u'_{t'}+u'u'_{x'}+w'u'_{y'} = -\frac{P_0}{\rho U^2}p'_{x'}+\frac{1}{Re}[\epsilon^2 u'_{x'x'}+u'_{y'y'}]+Fr, \quad (10)$$

$$\epsilon^2[w'_{t'}+u'w'_{x'}+w'w'_{y'}] = -\frac{P_0}{\rho U^2}p'_{y'}+\frac{1}{Re}[\epsilon^4 w'_{x'x'}+\epsilon^2 w'_{y'y'}], \quad (11)$$

$$u'_{x'}+w'_{y'} = 0, \quad (12)$$

with

$$u'=w'=0, \text{ at } y'=h'(x',t') \text{ and } u'=\alpha f(x',t'), \; w'=h'_{t'}, \text{ at } y'=0, \quad (13)$$

where $Re = \frac{\rho U H^2}{\mu L}$ is the Reynolds' number, $Fr = \frac{\rho g L}{\rho U^2}$ is the Froude number (ratio of inertial to gravitational forces), $\epsilon = \frac{H}{L}$ is the aspect ratio and $\alpha = \frac{f_0}{U}$ (ratio of axial stretch to axial velocity). The above equations also provide the characteristic pressure, $P_0 = \rho U^2$ (pressure due to inertial flow).

Hence, we have the following non-dimensional equations:

$$u'_{t'}+u'u'_{x'}+w'u'_{y'} = -p'_{x'}+\frac{1}{Re}[\epsilon^2 u'_{x'x'}+u'_{y'y'}]+Fr, \quad (14)$$

$$\epsilon^2[w'_{t'}+u'w'_{x'}+w'w'_{y'}] = -p'_{y'}+\frac{1}{Re}[\epsilon^4 w'_{x'x'}+\epsilon^2 w'_{y'y'}], \quad (15)$$

$$u'_{x'}+w'_{y'} = 0, \quad (16)$$

with the boundary conditions:

$$u'=w'=0, \text{ at } y'=0 \text{ and } u'=\alpha f(x',t'), \; w'=h'_{t'}, \text{ at } y'=h'(x',t'). \quad (17)$$

2.1 Varying bolus types

In this section we consider various regimes of Eq. (14)-(17) which correspond to different bolus types, e.g. liquid, partial liquid/solid and solid.

Case 1 - Inertia dominates : In this case we are considering a liquid bolus (e.g. water) where $Re \gg 1$ and $\epsilon = O(1)$ (i.e., $H \sim L$). In this case Eq. (14)-(17) reduce to

$$u_t + uu_x + wu_y = -P_x + Fr, \tag{18}$$
$$\epsilon^2[w_t + uw_x + ww_y] = -P_y, \tag{19}$$
$$u_x + w_y = 0, \tag{20}$$

with boundary conditions,

$$u = w = 0, \text{ at } y = 0 \text{ and } u = \alpha f(x,t), \;\; w = h_t, \text{ at } \; y = h(x,t). \tag{21}$$

Case 2 - Inertia and viscosity balance : This is equivalent to considering a semi-solid 'gloopy' type food, e.g. thick soup, very thick milkshake. Here $Re \sim 1$ and $\epsilon = O(1)$ (i.e., $H \sim L$), which leads to Eq. (14)-(17) reducing to:

$$u_t + uu_x + wu_y = -P_x + \frac{1}{Re}[\epsilon^2 u_{xx} + u_{yy}] + Fr, \tag{22}$$
$$\epsilon^2[w_t + uw_x + ww_y] = -P_y + \frac{1}{Re}[\epsilon^4 u_{xx} + \epsilon^2 w_{yy}], \tag{23}$$
$$u_x + w_y = 0, \tag{24}$$

with boundary conditions,

$$u = w = 0, \text{ at } y = 0 \text{ and } u = \alpha f(x,t), \;\; w = h_t, \text{ at } \; y = h(x,t). \tag{25}$$

Case 3 - Viscosity dominates : This is equivalent to swallowing a solid food which after mastication contains a high level of fluid. Here $Re \ll 1$ and $\epsilon = O(1)$ (i.e., $H \sim L$) leading to:

$$0 = -ReP_x + [\epsilon^2 u_{xx} + u_{yy}] + ReFr, \tag{26}$$
$$0 = -ReP_y + [\epsilon^4 u_{xx} + \epsilon^2 w_{yy}], \tag{27}$$
$$u_x + w_y = 0, \tag{28}$$

with boundary conditions,

$$u = w = 0, \text{ at } y = 0 \text{ and } u = \alpha f(x,t), \;\; w = h_t, \text{ at } \; y = h(x,t). \tag{29}$$

In the above equations, $ReFr = \dfrac{\rho g}{\mu U/H^2} = Bo$, where Bo is referred to as the Bond number (ratio of gravity to viscous forces). In this viscous dominated regime, one also re-scales the pressure, $\tilde{P} = ReP$. This is equivalent to using a characteristic pressure

$P_0 = \dfrac{\mu U/H}{\epsilon}$, which is the appropriate scaling for the viscous dominated regime. Hence, the above equation can be written as

$$0 = -\tilde{P}_x + [\epsilon^2 u_{xx} + u_{yy}] + Bo, \tag{30}$$
$$0 = -\tilde{P}_y + [\epsilon^4 u_{xx} + \epsilon^2 w_{yy}], \tag{31}$$
$$u_x + w_y = 0, \tag{32}$$

For the viscous dominant regime one can further simplify by assuming that the channel is long and thin, i.e., $\epsilon \ll 1$, and then apply lubrication theory to get an estimate of the flux through the channel. This is given by

$$q = ch_1 \left[\frac{P_{dn} - P_{up}}{12} - \frac{\left(\int_{\xi_{dn}}^{\xi_{up}} \frac{1}{f^2} d\xi + \frac{l_{up}+l_{dn}}{\epsilon^2} \right)}{\left(\int_{\xi_{dn}}^{\xi_{up}} \frac{1}{f^3} d\xi + \frac{l_{up}+l_{dn}}{\epsilon^3} \right)} \right] + ch_0$$

2.2 Numerical simulations

In order to understand how our model behaves we undertook numerical simulations of Eq. (14)-(17) using the finite element PDE solver COMSOL (COMSOL, Stockholm). In doing so we assumed the prescribed $h(x,t)$ to be given by

$$h(x,t) = H - (L - x) \tan \left(\theta_0 \sin \frac{\pi t}{T} \right), \tag{33}$$

where H is the average height of the hard palate, L is the length of the tongue, T the period of tongue movement and $\theta_0 = \arctan \dfrac{H}{L}$. Parameter values used are detailed in Table 1.

Simulation results showing the variation in the pressure field for swallowing a solid bolus are shown in Figure 3. Here we have set $Re = 0.1$ ($\mu = 0.1$). As can be seen in the respective figures, as the tongue moves upwards the pressure field increases such that the bolus is pushed from the left to the right of the region of interest as expected for a normal swallow. Similar results were obtained for a liquid ($Re = 1 \times 10^3$, $\mu = 0.1$) and semi-liquid bolus ($Re = 1$, $\mu = 10$).

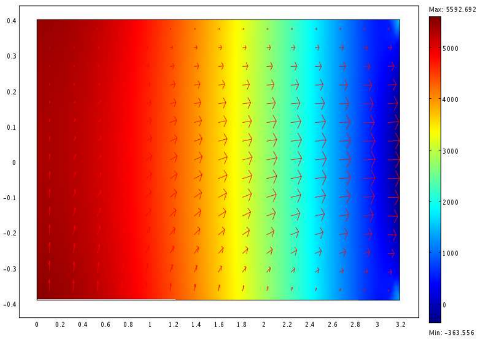
Numerical difficulties where found in obtaining further model solutions given the matching of the non-slip boundary condition on the left hand wall with the movement of the tongue - an issue to be resolved in future work.

3 Conclusions and future work

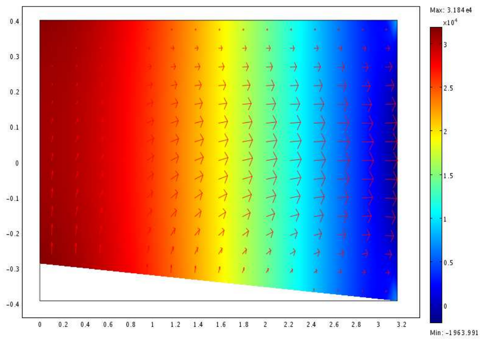
Normal swallowing is a complex dynamical process with different stages that are perfectly coordinated among them. During the Study Group the group managed to acquire a good understanding of the anatomical structures and of the dynamical process generally

Table 1: *Dimensional parameter values.*

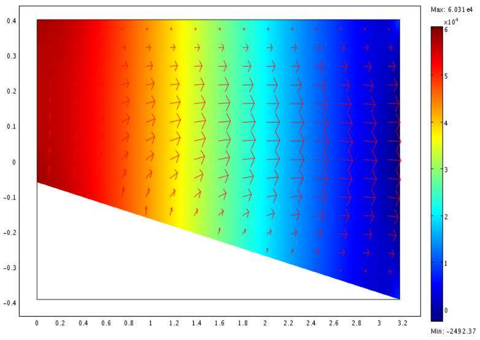
Parameter	Value	Description	Source
L	0.07m	Length from the front to the back of the tongue.	Estimated.
H	0.02m	Height of the average human palate.	Estimated.
ρ	1×10^3 kg/m^3	Bolus density.	For water.
μ	[0.01,100]kg/ms	Bolus viscosity.	Control parameter.
U	0.175m/s	Characteristic velocity.	Estimated.
T	0.4s	Period of the tongue movement.	Estimated.
A	0.2	Amplitude of the tongue movement.	Estimated.



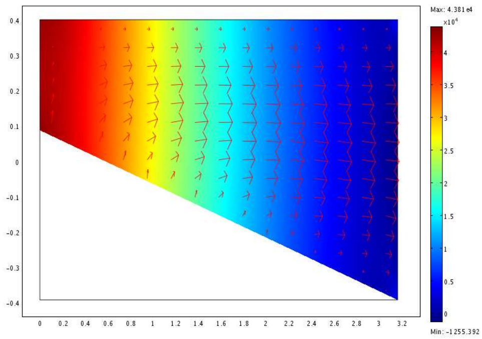
(a) Non-dimensional time - $t = 0.01$.



(b) Non-dimensional time - $t = 0.06$.



(c) Non-dimensional time - $t = 0.12$.



(d) Non-dimensional time - $t = 0.17$.

Figure 3: *(a)-(d) Numerical simulations of Eq. (14)-(17) showing the pressure field as a consequence of the prescribed movement of the tongue (bottom wall) towards the hard palate (upper wall). No flux conditions have been imposed on the left hand side in order to take into account the bolus being unable to escape the oral cavity in a normal swallow. Here we have considered the swallowing of a solid bolus with $Re = 0.1$ and the parameter values detailed in Table 1.*

involved in swallowing by constantly comparing and contrasting normal to abnormal cases (dysphagia). We developed a simple fluid dynamical model of the second and third stages of the swallow process, i.e. whereby a bolus is ejected from the oral cavity by the tongue. Various boli types were considered which allowed us to simplify the governing equations for liquid, semi-liquid and solid boli. Initial numerical simulations for these three cases have allowed us to determine how the pressure field varies within the oral cavity varies. Results for all three cases showed the bolus moving from the high pressure regions on the left of the region to the lower pressure on the right, indicating a normal swallow.

The major difficulty in mathematically modelling swallowing is the transition form one stage to another. The time coordination of the tongue muscles and the variation of the applied pressure variation by the tongue not only to propel the bolus but also to continuously modify the oral cavity are key elements that ensure a precise transition of the food bolus in normal swallowing. Using dimensional analysis and the data experimental data available we estimated that gravity plays a minor role in the bolus transition during the first three stages and that inertia and viscosity are the major terms in driving the bolus propulsion. Hence, it can be predicted that 'anti-gravitational' swallowing is possible, the only inconvenient being a discomfort in the larynx movements (rising and tilting) caused by the streching of the throat in an upside-down position. Guided by the Reynolds number values for different liquids ranging form water to honey and some solid aliments we concluded that normal swallowing benefits from strong and prefectly coordinated tongue movements that produce large forces and therefore can easily control inertia dominated bolus transitions of large Reynolds number liquids such as water. In contrast, abnormal swallowing with oral stage breakdown is characterised by weak and poor coordinated tongue movements restricting the patient suffering from dysphagia to intake only low Reynolds numbers liquids such as honey or thickened liquids for which the transitions regime is dominated by viscosity.

There is of course plenty of scope for future work since there is currently no full model to account for the different parameters that can vary during normal swallowing and to offer the possibilty to reproduce the whole process in a consistent manner. We had a quick look in the literature to check for existing models of swallowing, but an extensive literature review is definetely one of the next steps on the agenda. The mathematical attempts presented in the report were done in two spatial dimensions. However, future work would need to consider three-dimensional spatial extensions and also include wall curvature. Many suggestions have been discussed, but the predominant feeling was that modelling the tongue and the oral cavity as visco-elastic tube would maximise the chances of producing a good fluid mechanical model for the propulsion of the food bolus during normal swallowing. Numerical simulations involving complex geometrical structures to resemble the actual oral cavity and prescribed appropiate boundary conditions to account for the proper movement of the tongue are also required to test the hypothes we have made. This would help us to numerically asses the effect of the geometry of the structures involved in swallowing, the timing and pressure variations.

We believe that as with the case of other complex biological processes, mathematics can help in providing an understanding of the elements in the swallowing process. As a results of this, mathematicians can also propose some simplified models to account for the fundamental phenomena, but ultimately numerical simulations will be required to produce more accurate results which more accurately account for the clinical experimental data.

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